Physics 200: Fundamental Equations

Maxwell's Equations

Gauss's Law for Electric Field: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{\text{inside}}}{\varepsilon_0} = 4\pi k Q_{\text{inside}}$
Gauss's Law for Magnetic Field: $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$
Ampère/Maxwell: $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I_{encl} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ where $\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$
Faraday's Law: $\mathcal{E}_{\text{induced}} = \oint \vec{\mathbf{E}} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$ where $\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$

Fields, Forces, and Energy

Electric Force: $\vec{\mathbf{F}}^{\text{elec}}_{1 \text{ on } 2} = \frac{kQ_1Q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$; where $k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 (\text{N}\cdot\text{m}^2/\text{C}^2)$
Electric field: $d\vec{\mathbf{E}} = \frac{k\,dQ}{r^2}\hat{\mathbf{r}}$; $\vec{\mathbf{F}}^{\text{elec}}_{\text{on q}} = q\vec{\mathbf{E}}_{\text{at q}}$; $E_x = -\frac{dV}{dx}$
Electric potential: $V_A - V_B = -\int_B^A \vec{\mathbf{E}} \cdot d\vec{\ell}$; $dV = \frac{k\,dQ}{r}$
Electrostatic energy: $\Delta U_{\text{of q}} = q\Delta V$
Dielectrics: $E_{\text{in}} = E_{\text{applied}}/\kappa_E$; $\epsilon_{\text{in}} = \kappa_E \epsilon_0$; $C = \kappa_E C_0$
Magnetic Force: $d\vec{\mathbf{F}} = I d\vec{\ell} \times \vec{\mathbf{B}}$; $\vec{\mathbf{F}}^{\text{mag}}_{\text{on q}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$
Magnetic field: $d\vec{\mathbf{B}} = \frac{\mu_0 I d\vec{\ell} \times \hat{\mathbf{r}}}{4\pi r^2}$; $\mu_0 = 4\pi \times 10^{-7} (\text{T}\cdot\text{m/A})$;
Magnetic dipole: $\vec{\mu} = NI\hat{\mathbf{n}}A$; *Torque*: $\vec{\tau} = \vec{\mu} \times \vec{\mathbf{B}}$; $U_{\text{dipole}} = -\vec{\mu} \cdot \vec{\mathbf{B}}$

Circuits

Capacitors: $C = Q/\Delta V$; $U_{\text{E}} = \frac{1}{2}C(\Delta V)^2$
Power: $P = I\Delta V$;
Resistors: $I = \frac{dQ}{dt} = \frac{\Delta V}{R}$; $dR = \frac{\rho\,dL}{A}$
Series/Parallel: $C_{\text{series}}^{-1} = \sum_i C_i^{-1}$; $C_{\text{parallel}} = \sum_i C_i$; $R_{\text{series}} = \sum_i R_i$; $R_{\text{parallel}}^{-1} = \sum R_i^{-1}$
Kirchhoff's Laws: $\sum_{\text{Closed loop}} \Delta V_i = 0$; $\sum I_{\text{in}} = \sum I_{\text{out}}$
RC: $Q(t) = Q_{\text{final}}\left(1 - e^{-t/RC}\right)$; $Q(t) = Q_{\text{initial}} e^{-t/RC}$
Inductors: $\mathcal{E}_{\text{ind}} = -L\frac{dI}{dt}$; $L = N\frac{\Phi_B}{I}$; $U_{\text{B}} = \frac{1}{2}LI^2$; $M_{12} = \Phi_{\text{in 2}}/I_{\text{in 1}}$; $\mathcal{E}_1 = -M_{12}\frac{dI_2}{dt}$
LR: $I(t) = I_{\text{final}}\left(1 - e^{-t/(L/R)}\right)$; $I(t) = I_{\text{initial}} e^{-t/(L/R)}$;

Electromagnetic Waves and Field Energy Density

Field Energy Density: $u_E = \frac{1}{2}\varepsilon_0 E^2$; $u_B = \frac{1}{2\mu_0}B^2$; *Momentum:* $p = U/c$
Waves: $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3 \times 10^8$ m/s; $c = \lambda f$; $k = 2\pi/\lambda$; $\omega = 2\pi f$
Poynting vector: $\vec{\mathbf{S}} = \frac{1}{\mu_0}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$; *Intensity*: $I = |\mathbf{S}|_{avg} = c\frac{1}{2}\varepsilon_0 E_{\text{m}}^2 = \frac{P_{\text{av}}}{A}$; $cB = E$

Optics

Reflection/Refraction: $c_1 = c/n_1$; $\theta_{\text{inc}} = \theta_{\text{ref}}$; $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$
Interference: Constructive: $\delta = 2\pi m$; Destructive: $\delta = 2\pi(m + 1/2)$
Phase shift of reflected wave: $n_1 \to n_2$, 180^o if $n_1 < n_2$; 0^o otherwise

Additional Information

Electric fields: $E_{\text{infinite sheet}} = \frac{\sigma}{2\varepsilon_0}$; $E_{\text{infinite line}} = 2k\lambda/r$; $E_{\text{conducting plate}} = \frac{\sigma}{\varepsilon_0}$; $C_{\text{parallel plate}} = \frac{\varepsilon_0 A}{d}$
Magnetic fields: $B_{\text{infinite wire}} = \frac{\mu_0 I}{2\pi r}$; $B_{\text{solenoid}} = \mu_0 n I$; $L_{\text{solenoid}} = \mu_0 n^2 A l$; $B_{\text{current loop}} = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$